15MAT21

Second Semester B.E. Degree Examination, July/August 2022 Engineering Mathematics – II

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Solve
$$(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$$
.

(05 Marks)

b. Solve
$$\frac{d^2y}{dx^2} - 4y = \cosh(2x - 1) + 2^x$$
.

(05 Marks)

c. Solve
$$y'' - 2y' + y = e^x \log x$$
 by the method of variation of parameters.

(06 Marks)

OR

2 a. Solve
$$(D^2 - 2D + 4)y = e^x \cos x$$
.

(05 Marks)

b. Solve
$$(D^2 + 4)y = x^2 + \sin 2x$$
.

(05 Marks)

(06 Marks)

c. Solve
$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x + \sin x$$
 by the method of undetermined coefficients.

Module-2

3 a. Solve
$$x^2y'' + xy' + y = 2\cos^2(\log x)$$
.

(05 Marks)

b. Solve
$$y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0$$
.

(05 Marks)

c. By reducing into Clairaut's form, obtain the general and singular solution of
$$xp^3 - yp^2 + 1 = 0$$
. (06 Marks)

OR

4 a. Solve
$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$$

(05 Marks)

b. Solve for y:
$$x^2p^4 + 2xp - y = 0$$
.

(05 Marks)

c. Solve for x :
$$P = tan \left[x - \frac{P}{1 + P^2} \right]$$
.

(06 Marks)

Module-3

5 a. Obtain the partial differential equation by eliminating the arbitrary function given $\frac{7}{3} + \frac{2}{3} \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right)$

 $Z = y^2 + 2f\left(\frac{1}{x} + \log y\right).$

(05 Marks)

b. Solve
$$\frac{\partial^2 z}{\partial y^2} = z$$
 given that when $y = 0$, $z = e^y$ and $\frac{\partial z}{\partial y} = e^{-x}$.

(05 Marks)

(06 Marks)

c. Derive one dimensional wave equation $\frac{\partial^2 u}{\partial t_{\star}^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

(06 Marks)

- Obtain the partial differential equation of $\phi(x + y + z, x^2 + y^2 z^2) = 0$. (05 Marks)
 - b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given that $\frac{\partial z}{\partial y} = -2\sin y$ when x = 0 and z = 0 if $y = (2n + 1)\frac{\pi}{2}$. (05 Marks)
 - Find the solution of heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. (06 Marks)

- 7 a. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2) dz dy dx$. (05 Marks)
 - Change the order of integration and evaluate $\int_{0}^{\infty} \frac{e^{-y}}{y} dy dx$. (05 Marks)
 - Obtain the relation between Beta and Gamma function $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (06 Marks)

- Find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration. (05 Marks)
 - Find the volume of the tetrahedron bounded by the planes x = 0, y = 0, z = 0, $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.
 - Show that $\int_{0}^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \times \int_{0}^{\pi/2} \sqrt{\sin \theta} d\theta = \pi.$ (06 Marks)

- Find the Laplace transform of $\frac{\cos at \cos bt}{t}$ (05 Marks)
 - If f(t) is a periodic function of period T, then prove that $L[f(t)] = \frac{1}{1 e^{-ST}} \int_{0}^{t} e^{-st} f(t) dt$.
 - (05 Marks) Find the inverse Laplace transform of $\frac{4s+5}{(s+1)^2(s+2)}$

- Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \end{cases}$ interms of unit step function and hence find its $\cos 3t, & t > 2\pi \end{cases}$
 - Laplace transform. (05 Marks)
 - b. Find $L^{-1} \left[\frac{1}{(s-1)(s^2+1)} \right]$ by using convolution theorem. (05 Marks)
 - c. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-t}$ with y(0) = 0, y'(0) = 0. (06 Marks)